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THE DETERMINATION OF THE CONFIGURATION OF A CRACK IN AN ANISOTROPIC MEDIUM†

A. O. VATUL'YAN

Rostov-on-Don

e-mail: vatulyan@aaanet.ru

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Inverse geometric problems of the anisotropic theory of elasticity for bodies with cracks of arbitrary configuration are investigated using a linear approach. Questions of the uniqueness of solution of the inverse problems that arise are studied, and effective schemes are constructed for solving them by a combination of the boundary element method and a regularized iteration procedure. The example of the reconstruction of a rectilinear crack in an orthotropic layer is examined. © 2004 Elsevier Ltd. All rights reserved.

The development of effective mathematical models of the diffraction of elastic waves by cracks is an extremely pressing problem in the development of ultrasonic methods for revealing internal and surface defects (cavities and cracks) with a subsequent determination of their characteristic dimensions and configuration from the displacement field measured on the boundary of the body. If the size of the defect is commensurate with the wavelength of the probing field or less than it, the use of reliable mathematical models becomes particularly important by virtue of the fact that, in the case, the field recorded on the body surface changes very little when there is a defect present when solving the inverse geometric problems of the theory of elasticity that arise in this case, a strict approach is necessary, based on a fairly accurate solution of the corresponding boundary-value problems of the dynamic theory of elasticity. The most popular model of a crack is currently the mathematical cut, in which the displacements undergo finite jumps on a certain surface, and the crack surfaces are open and do not interact [1]. This concept historically stems from formulations for static problems of the theory of elasticity, and here the jumps are determined either from the condition for there to be no stresses on the crack surfaces (within the framework of the superposition principle) from the condition for the normal stresses to be constant on them.

One of the most effective methods of investigating direct and inverse problems of elastic bodies with defects under conditions of steady vibrations is the method of reduction to systems of boundary integral equations (BIEs), which enables the dimension of the direct problems to be reduced and enables a system of non-linear operator equations for solving the inverse problems to be formulated. An approach of this kind for defects of the cavity type in an unbounded medium and when there is a rectilinear boundary was proposed earlier (see, for example, [2, 3]). As regards the procedure for determining the configuration of cracks in a solid body from information on physical fields at the boundary of the body, in recent years investigations have been carried out in the following three areas:

(1) a study of inverse problems for Laplace's equation and the modelling of the procedure for identifying the crack by studying the features of the structure of either the thermal or the electrostatic fields in bodies with defects [4–8] using a certain "non-reciprocity" functional;

(2) the reconstruction of a crack in an infinite medium from the radiation patterns of elastic waves (similar to approaches described earlier [2]) in the far zone [9–11] and the formulation of systems of BIEs of the first kind;

(3) a study of the positioning of cracks situated at the interface of two elastic materials [12], combining a procedure for solving the direct problem by the finite element method and solving the problem of field continuation.

Within the framework of the isotropic theory of elasticity, the BIE method was used to solve a broad class of problems of the vibrations of elastic bodies with cracks without the interaction of the surfaces of the crack. Methods have been developed [13] that enable the vibrations of bodies with a single crack to be studied, as well as the vibrations of bodies with a system of parallel cracks in a half-space of a layer, and which enable the solution of integral equations to be obtained in semi-analytical form, without

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requiring lengthy computing time. However, if the crack is inclined to the rectilinear boundary, or is not a plane crack, the only effective means of investigating the direct and inverse problems of the crack theory is the general BIE method, and, in the case of a numerical realization, the boundary element method based on it.

Note that, in many cases, to construct an adequate model of the reflection of elastic waves from a crack, account must be taken of the anisotropy possessed by many actual structural materials and alloys, and also rocks, which considerably complicates the calculation of the wave fields reflected from the defect.

1. FORMULATION OF THE PROBLEM

Consider the steady vibrations, with frequency ω, of an elastic anisotropic body V bounded by a piecewise-smooth surface $S = S_1 \cup S_2$. The vibrations are caused by a load p_i applied to parts of the boundary $S_{20} \subset S_2$, while a part of the boundary S_1 is restrained. We will assume that the body V is weakened by a crack on the smooth internal bilateral surface $S_0^{\mp}$ at which the components of the displacement vector undergo jumps $\chi_i = u_i|_{S_0^+} - u_i|_{S_0^-}$; we will also assume that, during the vibrations, the crack surfaces do not interact. The boundary-value problem has the form

$$\sigma_{ij,\,j} + \rho\omega^2 u_i = 0, \quad \sigma_{ij} = c_{ijkl}u_{k,\,l} \tag{1.1}$$

$$\sigma_{ij}n_j|_{S_2} = p_i, \quad u_i|_{S_1} = 0, \quad \sigma_{ij}n_j^{\pm}|_{S_0^{\pm}} = 0 \tag{1.2}$$

where the loads p_i are non-zero on S_{20}. Here, c_{ijkl} are tensor components of the constants of elasticity, satisfying the normal conditions of symmetry and positive determinacy, and $n_j^{\mp}$ are the components of the unit vectors of the normals to the surfaces $S_0^{\pm}$. The inverse problem of identifying the crack consists of finding the form of the surface S_0^+ from the prescribed (measured) displacement field on the part of the boundary S_{21} free from loads ($S_{20} \cap S_{21} = \varnothing$) from the condition

$$u_i|_{S_{21}} = g_i \tag{1.3}$$

Remark. When the interaction of the crack surfaces are taken into account, it is necessary, in the boundary conditions on the crack, to allow for the process of the crack surfaces closing up during vibrations; however, in this case the direct problem immediately becomes non-linear. These conditions can be formulated in the form of the inequality $\chi_n = \chi_j n_j^+ \geq 0$, where, if the inequality is strict, we also have the condition $\sigma_{ij}n_j^+|_{S_0^{\pm}} = 0$. But if, on some section of the crack, $\chi_n = 0$ (the crack surfaces are closed up), then, along this section of the crack, the condition $\sigma_{ij}n_j^+n_j^- < 0$ must be satisfied, which corresponds to its compression. Here it turns out that, by virtue of the non-linearity of the boundary conditions, it is not possible to study steady vibrations, and it is necessary to examine the problem in a non-stationary formulation.

2. THE UNIQUENESS OF THE SOLUTION OF THE PROBLEM OF RECONSTRUCTING THE CRACK

As is will known, uniqueness is a key problem in investigating inverse geometric problems. When a body V is weakened by a cavity, the scheme of proof of the uniqueness theorem is similar to schemes of proof in the acoustic case [2]. Note that uniqueness theorems were formulated within the framework of the first [4] and second areas [7, 9]. In the case of cracks in a bounded anisotropic elastic body, the problem of proving the uniqueness of its identification requires different mathematical tools.

We will formulate the conditions which ensure the uniqueness of the solution of the inverse problem of reconstructing the surface S_0^+.

Theorem. Suppose that, under the conditions of the formulation of inverse problem (1.1)–(1.3), the data of (1.3) are known in a certain segment of variation of the frequency $\omega \in [\omega_1, \omega_2]$ in the non-resonance region. Then the problem of finding the surface S_0^+ has a unique solution in the class C^2 of smooth surfaces with a smooth edge that are contained strictly within the volume V.

We will prove this assertion assuming that two solutions of the inverse problem exist, characterized by displacement vectors with components $u_j^{(1)}$ and $u_j^{(2)}$ and surfaces $S_0^{(1)}$ and $S_0^{(2)}$.

We will introduce into consideration a vector with the components $v_j = u_j^{(1)} - u_j^{(2)}$ and a tensor with the components $T_{ij} = c_{ijkl}v_{k,l}$. Homogeneous equations of motion

$$T_{ij,j} + \rho\omega^2 v_i = 0 \tag{2.1}$$

exist in the region $V_{12} = V(S_0^{(1)} \cup S_0^{(2)})$, and homogeneous boundary conditions are

$$v_i|_{S_1} = 0, \quad T_{ij}n_j|_{S_2} = 0, \quad v_i|_{S_{21}} = 0 \tag{2.2}$$

Then, for an elliptical operator of the theory of elasticity (2.1), as shown earlier [14], we actually have a Cauchy problem with zero data on S_{21} and, by virtue of the uniqueness of its solution, $v_i \equiv 0$ throughout the region V_{12} up to its boundary; in particular

$$T_{ij}n_j^{(2)}|_{S_0^{(2)}} = 0 \tag{2.3}$$

Below we will examine two cases.

The case $S_0^{(1)} \cap S_0^{(2)} = \varnothing$. We will introduce into our consideration the region $V_0^{(2)} \subset V$ containing $S_0^{(2)}$ strictly within it, such that $V_0^{(2)} \cap S_0^{(1)} = \varnothing$. Within this region, the equations of motion

$$\sigma_{ij,j}^{(1)} + \rho\omega^2 u_i^{(1)} = 0, \quad \sigma_{ij}^{(1)} = c_{ijkl}u_{k,l}^{(1)} \tag{2.4}$$

are satisfied, and also the condition

$$\sigma_{ij}^{(1)}n_j^{(2)}|_{S_0^{(2)}} = 0 \tag{2.5}$$

for all $\omega \in [\omega_1, \omega_2]$ by virtue of relation (2.3). Since the solutions of the elliptical system (2.4) $u_i^{(1)}$ are analytical functions of the coordinates (and the frequency ω) [15] in the region $M = V_0^{(2)}[\omega_1, \omega_2]$, then, by virtue of the fact that condition (2.5) is satisfied on a certain hypersurface $S_0^{(2)}[\omega_1, \omega_2]$ lying within M, the function $u_i^{(1)}$ is identically equal to zero in the region $V_0^{(2)}$ and continues to be analytically zero up to the boundary S_{21}, which contradicts boundary condition (1.3).

The case $S_0^{(1)} \cap S_0^{(2)} \neq \varnothing$. In this case the region $V_0^{(2)} \subset V$ is selected in such a way that it contains the part $S_1^{(2)} \subset S_0^{(2)}$, where $S_1^{(2)} \cap S_0^{(2)} = \varnothing$.

The subsequent reasoning follows that of the first case.

The theorem is proved.

3. FORMULATION OF THE SYSTEM OF OPERATOR EQUATIONS

At the first stage of investigating the problem of identification, a solution is constructed for the direct problem of calculating the wave fields in an elastic body weakened by a crack of known configuration S_0^+. For this, we use the main ideas of potential theory, which enables us to reduce the initial boundary-value problem to a system of integral equations in terms of the jumps of displacements on the crack that were introduced above. The most effective approach involves first reducing the initial problem (1.1), (1.2) for a body with a crack within the framework of the theory of dislocations [16] to a system of equations of the theory of elasticity with fictitious mass forces $f_i = -[c_{ijkl}n_k^+\chi_l\delta(\zeta)]_{,j}$ for a homogeneous body V. Then, on the basis of the reciprocity theorem for an elastic anisotropic body, the field of elastic displacements within $V(\xi \in V)$ can be found by means of Somigliana's formulae [1]

$$u_m(\xi) = \int_S \sigma_{ij}n_jU_i^{(m)}(x,\xi)dS_x - \int_S \sigma_{ij}^{(m)}(x,\xi)n_ju_idS_x + \int_V U_i^{(m)}(x,\xi)f_idV_x \tag{3.1}$$

where $U_i^{(m)}(x, \xi)$ and $\sigma_{ij}^{(m)}(x, \xi)$ are respectively the fundamental and singular solutions for an anisotropic medium; their explicit respresentations cannot be constructed, but it is possible to construct their integral representations, which is quite sufficient for the numerical implementation of the boundary element method (as was pointed out, for example, in [17]).

Taking into account the expression for f_i and selecting in Eq. (3.1), as $U_i^{(m)}(x, \xi)$, Green's matrix function for the operator of the anisotropic theory of elasticity (1.1) with the boundary conditions

$$U_i^{0(m)}(x, \xi)\big|_{S_1} = 0, \quad \sigma_{ij}^{0(m)}(x, \xi)n_j\big|_{S_2} = 0 \tag{3.2}$$

we obtain the following representation for calculating the displacement fields inside the body V

$$\begin{aligned} u_m(\xi) &= u_m^s(\xi) + \int\limits_{S_0^+} \sigma_{kl}^{0(m)}(x, \xi)\chi_l(x)n_k(x)dS_x \\ u_m^s(\xi) &= \int\limits_{S_{20}} p_i(x)U_i^{0(m)}(x, \xi)dS_x \end{aligned} \tag{3.3}$$

where $u_m^s(\xi)$ is the field in the medium without a defect (the reference field). If the expansion functions are known, then, using formulae (3.3), it is possible to calculate the displacement field throughout the region, including on the boundary S_2. To determine the functions of the crack opening by the normal method, a system of boundary equations is formulated by satisfying the boundary conditions on the crack (1.2):

$$K\chi = \int\limits_{S_0^+} k_{jl}(x, y)\chi_l(x)dS_x = F_j(y), \quad y \in S_0^+ \tag{3.4}$$

The kernels $k_{jl}(x,y)$ are hypersingular and have a second-order moving singularity, and the corresponding integrals are understood in the sense of a Hadamard finite value [18]; the functions $F_j(y)$ are expressed in terms of the reference displacement field.

Using relations (3.3) and (3.4) and the solution of the direct problem, at the second stage of the investigation, in the inverse problem, taking into account condition (1.3), a system of operator equations is formulated in $\chi_l(x)$ and S_0^+

$$\begin{aligned} &\int\limits_{S_0^+} k_{ml}^0(x, \xi)\chi_l dS_x = g_m^*(\xi) = g_m(\xi) - u_m^s(\xi), \quad m = 1, 2, 3, \quad \xi \in S_{21} \\ &\int\limits_{S_0^+} k_{jl}(x, y)\chi_l(x)dS_x = F_j(y), \quad j = 1, 2, 3, \quad y \in S_0^+ \end{aligned} \tag{3.5}$$

where $k_{ml}^0(x, \xi) = \sigma_{kl}^{0(m)}(x, \xi)n_k(x)$. The integral operators in the system of BIEs (3.5) extend only to the boundary of the crack, which significantly reduces the volume of computational work when solving both the direct and the inverse problem. If the fundamental solutions of the corresponding operators occur as kernels, the system of BIEs contains a number of intermediate unknowns, in particular the displacements on the body surface. Unfortunately, the construction of Green's functions, even in the form of integral representations, only encounters no difficulties for canonical regions in the form of a layer, an infinite cylinder, and a half-space.

Note that the system of operator equations (3.5), along with hypersingular operators, also contains Fredholm operators of the first kind with smooth kernels, since $S_{21} \cap S_0^+ = \varnothing$. By virtue of this, the problem of finding S_0^+ from system (3.5) is ill-posed and, consequently, unstable to small disturbances of the prescribed functions $g_j(x)$, which is characteristic of inverse geometric problems. Because of this, the procedure for a numerical investigation of system (3.5) requires regularization [19] in a particular form.

4. A METHODS OF SOLVING THE PROBLEM OF IDENTIFICATION

The most effective scheme for a numerical analysis of non-linear system (3.5) consists of two stages, as described below.

Stage 1. In the first stage, the method for finding the simplest configuration S_0^+ is based on using the principle of regularization on compact sets, in particular on the preliminary parameterization of this surface by describing it with a finite number of parameters c_k. For example, for a plane elliptical crack, there are seven parameters of this type (the coordinates of the centre, the components of the normal vector, and the semi-axes of the ellipse); for a tunnel crack perpendicular to the boundary of the layer there are, in all, two parameters of this kind: the length of the crack and the distance from the nearest tip of the crack to the boundary of the layer. Note that, as the parameters to be determined, it is best to select certain invariant characteristics of the crack that are not related to the selection of the system of coordinates, such as the area, the length of the crack contour, the distance to the free boundary, etc. The method for finding these parameters is based on a discrete representation of the integral operators in system (3.5) in terms of the nodal values of the functions of the crack opening and on determining the parameters c_k from the condition for a minimum of the non-quadratic discrepancy functional $\Phi(c_k)$ generated by the second relation of (3.5). In finding the minimum discrepancy functional, iteration schemes are usually employed.

The implementation of this approach requires repeated solution of the direct problem (1.1), (1.2), which is carried out using the simplest version of the boundary element method. In this case, the solution of the system of hypersingular equations (3.4) reduces to solving a system of linear algebraic equations in the nodal values of χ_{lq} (with fixed values of the parameters c_k)

$$\sum_{q=1}^{N} A_{jlqp}\chi_{lq} = F_{jp}, \quad p = 1, 2, \ldots, N \tag{4.1}$$

$$A_{jlqp} = \int_{S_q} k_{jl}(x, y_p)dS_x, \quad F_{jp} = F_j(y_p)$$

$\cup_{q=1}^{N} S_q$ is the approximation of the surface S_0^+ by a polyhedron with triangular faces, and y_p is the centre of gravity of the triangle S_p (in the plane case $n = 2$, the contour of the crack is approximated by an N-section broken line, and y_p is the middle of the pth link).

Stage 2. In the second stage, the crack configuration is determined more precisely using the linearization method. The system of operator equations is linearized in the vicinity of the simplest configuration found in the first stage, with subsequent discretization using a combination of the ideas of the boundary element method and Tikhonov's regularization method [19].

We will find the surface S_0^+ on a set of smooth surfaces of class C^2, star surfaces with respect to a certain centre, where we will assume that the boundary ∂S_0^+ of the surface S_0^+ is a smooth curve. We will map S_0^+ onto a segment of a unit sphere Q_1 which is specified by the equation $x = R(u)$, $u = (u_1, u_2) \in Q_1$.

Suppose the vector function $R_0(u)$ corresponds to a plane round crack found in the first stage. We will linearize the first of the integral equations of system (3.5) in the vicinity of $R_0(u)$, introducing $z(u) = R(u) - R_0(u) = \{z_j(u)\}, j = 1, 2, 3$ and using the following expansions

$$\begin{aligned} \sigma_{kl}^{0(m)}(R(u), \xi) &= \sigma_{kl}^{0(m)}(R_0(u), \xi) + \sigma_{kl,j}^{0(m)}(R_0(u), \xi_1)z_j(u) + o(z(u)) \\ \chi_l(R(u)) &= \chi_l(R_0(u)) + \chi_{l,j}(R_0(u))z_j(u) + o(z(u)) \\ n_k^+(x)dS_x &= \ni_{kst}[R_{so,1}(u)R_{to,2}(u) + R_{so,1}(u)z_{t,2}(u) + R_{to,2}(u)z_{s,1}(u) + o(\nabla z(u))]du_1du_2 \end{aligned} \tag{4.2}$$

(here $\ni_{kst}$ are the Levi–Civita symbols). The linearization procedure gives the following system of Fredholm integral equations of the first kind with smooth kernels

$$L_{mj}z_j = G_m, \quad m = 1, 2, 3$$

$$\begin{aligned} L_{mj}z_j = \int_{Q_1} \ni_{kjt}\Big[&\sigma_{kl,j}^{0(m)}(R_0, \xi)\chi_l(R_0) + \sigma_{kl}^{0(m)}(R_0, \xi)\chi_{l,j}(R_0) - \\ &-\Big(\frac{\partial R_{t0}}{\partial u_2}\frac{\partial}{\partial u_1} - \frac{\partial R_{t0}}{\partial u_1}\frac{\partial}{\partial u_2}\Big)(\sigma_{kl}^{0(m)}(R_0, \xi)\chi_l(R_0))\Big]z_j(u_1, u_2)du_1du_2 \end{aligned} \tag{4.3}$$

Here $G_m(\xi) = g_m^*(\xi) - g_m^0(\xi)$, and $g_m^0(\xi)$ corresponds to the configuration $R_0(u)$.

After solving system (4.3) by the method of regularization and finding the functions $z_j(u_1, u_2)$, it is possible to change to a new crack configuration and then construct a linearization in its vicinity using relations (3.4) and (4.3).

5. NUMERICAL EXPERIMENT

To illustrate the proposed approach, we will examine the problem of reconstructing a transverse crack in an orthotropic strip of thickness H with a rigidly restrained lower face. Vibrations in the strip are caused by a normal load applied to its upper face in a finite interval, while the remainder of the boundary is load-free. The formulation of the problem of the radiation condition is closed, in the formulation of which use is made of the principle of limit absorption [20]. The coordinates of the crack tips are determined from the known displacement field on part of the free boundary.

The representation of the wave field in the strip was found by constructing Green's matrix function for the strip with boundary conditions of type (3.2), which is constructed by means of the Fourier integral transform and can be represented in the form of single integrals over a certain contour σ in the complex plane, selected in accordance with the principle of limit absorption. The system of hypersingular integral equations (3.3) in the jumps χ_j in the case considered of a transverse crack splits into two independent equations

$$\int_a^b \chi_j(\xi_3) k_{jj}(\xi_3, x_3) d\xi_3 = F_j(x_3), \quad j = 1, 3, \quad x_3 \in [a, b] \tag{5.1}$$

where their kernels can be represented in the form

$$k_{jj}(\xi_3, x_3) = \int_\sigma \frac{D_{jj}(a_1, \xi_3, x_3)}{D(\alpha_1)} d\alpha_1 \tag{5.2}$$

and $D_{jj}(\alpha, \xi_3, x_3)$ and $D(\alpha_1)$ are known functions of their arguments, which on account of their length are not given here. Unlike the case examined earlier [13], the kernels $k_{jj}(\xi_3, x_3)$ are non-difference kernels but have a moving singularity at $x_3 = \xi_3$.

Boundary integral equations of the form (5.1) and (5.2) were solved using the boundary element method, which, for these types of equations, was described earlier [21]. The displacement field calculated from representation (3.3) on the upper face of the layer at points x_1^j served as the initial data when solving the inverse problem.

The inverse problem consisted of finding the crack tips a and b from the known displacements u_k^{0j}, specified at the points x on part of the boundary $x_3 = H$, and reduced to the simultaneous solution of the system of boundary integral equations (5.1) and minimization of the corresponding discrepancy functional

$$\Phi(a, b) = \sum_j \left| u_k(x_1^j, H) - u_k^{0j} \right|^2$$

In the numerical realization of the proposed approach, the boundary integral equations were reduced to a system of non-linear equations in the unknown parameters of the crack a and b and the nodal values χ_k^j. An iteration process was constructed for determining the values of a and b, and here either an internal crack of maximum size or the crack configuration to which the values of the parameters which give the minimum discrepancy functional correspond when it is on a certain uniform grid within the search triangle $0 \le a < b < H$ is selected as the initial approximation.

As an example of the reconstruction, below we give the results of calculations when solving a model problem for an orthotropic strip of austenite steel weakened by a transverse crack, the coordinates of the tips of which are $\theta_0 = a/H = 0.7$, $\theta_1 = b/H = 0.95$ with different values of dimensionless frequency $\kappa = H\omega\sqrt{\rho/c_{33}} \in [3.3, 5.3]$, when there are a different number of radiating modes in the layer. Note the effectiveness with which the crack tips to be identified can be found, as illustrated in Fig. 1, which shows graphs of the reconstruction of the coordinates of the crack (θ_0, θ_1) as a function of the number of iterations n for different values of the parameter κ. The results of numerical experiments using the proposed procedure showed that, for low and medium frequencies, to determine only the size of the

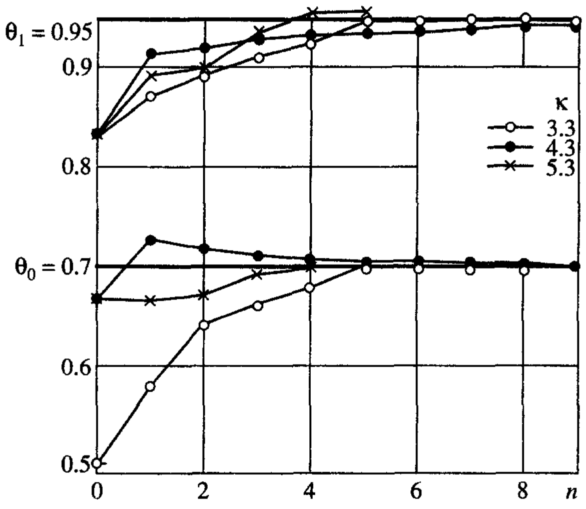

Fig. 1

deepened crack with an error of the order of 1%, 5–8 iterations are sufficient, while identification of the coordinates of its ends requires 4–12 iterations.

It should be noted that, when the size of the crack is reduced compared with the wavelength, the discrepancy functional has multiple local minima, and in this range of frequencies it is necessary to use finer means of minimization. For a near-surface crack, when the frequency of the vibrations decreases, the resolution of the method deteriorates. After a certain number of iterations, the calculated values of the parameters a and b are observed to approach the accurate values slowly; when the vibration frequency and the number of radiating modes increase, the rate of convergency of the proposed method of reconstruction increases. When there is a small number of points at which the displacement field x_1^j is recorded, it is occasionally possible for a phantom mirror image of the crack to appear with respect to the middle surface of the layer, but in this case its sizes is reconstructed correctly. To eliminate the phantom solution, either additional information on the field with a larger number of recording points x_1^m was used or information on the displacement field when the vibration frequency changes in accordance with the uniqueness theorem. By taking into account the additional information it was possible to eliminate the phantom solution.

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